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Project 2
AE 3330 B Aerospace Vehicle Performance
Summer 2022

PART I

You're piloting an X-Wing fighter in orbit around Alderaan with a radius of 6,400km and a mass of 10,000kg. Suddenly the Death Star appears out of hyperspace and inserts itself into an orbit with a radius of 11,000km. The Death Star has a mass of $2.5x10^{18}$ kg, Alderaan has a mass of $5x10^{24}$ kg. Note both the X-Wing and the Death Star are in the Z-Y plane with the X-Wing only having a radius component in the Z direction and the Death Star only in the Y direction. The figure below displays the orbit geometry.

a. Using the two-body formulation, find the acceleration of the X-Wing relative to Alderaan prior to the arrival of the Death Star.

The acceleration of the X-wing relative to Alderaan prior to the arrival of the Death Star is about **8.147 m/s^2**. See MATLAB code below:

```
%Mass of the X-wing
m1 = 10000; %kg
%Mass of Alderaan
m2 = 5*10^24; %kg
%Mass of the Death Star
m3 = 2.5*10^18; %kg

%radius of x-wing to origin
r1 = [0 0 6400*1000]; %m in the z direction, in the z-y plane
%radius of alderaan to origin
r2 = [0 0 0]; %m jk alderaan IS the origin
%radius of death star to the origin
r3 = [0 11000*1000 0]; %m in the y-direction

G = 6.674*10^-11; %Nm^2 / kg^2

%vector between alderaan and xwing
r12 = r2 - r1;
r12_hat = r12 ./ norm(r12);
r21 = r1 - r2;
r21_hat = r21 ./ norm(r21);

%2-body problem
rdd = (-G*(m1+m2)*r12_hat)/norm(r12)^2;
```

b. After the Death Star arrives, use the N-body formulation to find the acceleration of the X-Wing relative to Alderaan. Compare your answer to that of part A, ie compute % difference assuming the N-body formulation is the accepted answer.

The new acceleration of the X-Wing relative to Alderaan is almost the same number, with the later digits changed. A longer version of the 2-body solution is: **8.146972656250000 m/s^2** in the negative z direction. The longer version of the N-body solution is **8.146973174327263 m/s^2** in the negative z direction with a component even smaller in the positive y direction:
4.885×10^{-07} m/s^2.
The percent difference between the two, assuming the N-body formulation, is a**: 6.360×10^{-6}** percent difference.

See work MATLAB code below:

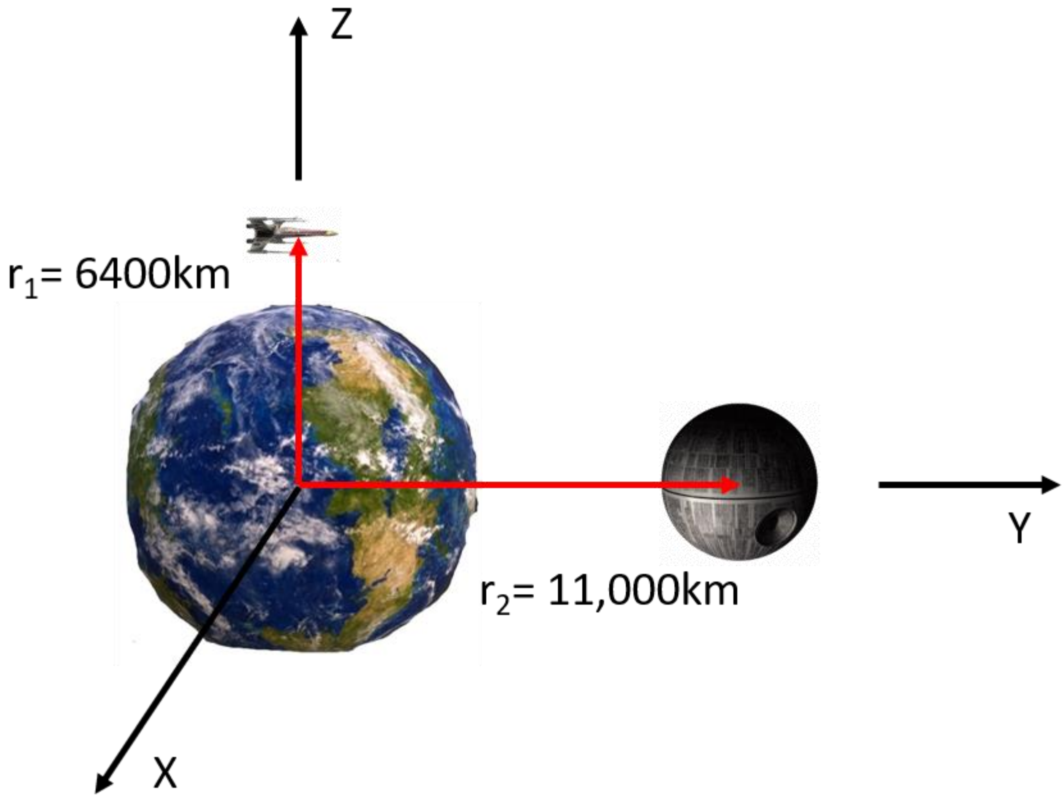
```
%vector between death star and x-wing
r31 = r1 - r3;
r31_hat = r31 ./ norm(r31);
r13 = r3 - r1;
r13_hat = r13 ./ norm(r13);

%vector between Death Star and Alderaan
r32 = r2 - r3;
r32_hat = r32 ./ norm(r32);
r23 = r3 - r2;
r23_hat = r23 ./ norm(r23);

r1dd = -G*(m2*r21_hat/(norm(r21)^2) + m3*r31_hat/(norm(r31)^2));
r2dd = -G*(m1*r12_hat/(norm(r12)^2) + m3*r32_hat/(norm(r32)^2));
r3dd = -G*(m1*r13_hat/(norm(r13)^2) + m2*r23_hat/(norm(r23)^2));

%motion of x-wing wrt Alderaan
r12dd = r2dd-r1dd;

%motion of x-wing wrt Death Star
r13dd = r3dd-r1dd;

% Percent difference
percent_difference = (norm(r12dd) - norm(rdd)) / norm(r12dd) * 100
```

c. Using the N-body formulation, find the acceleration of the X-Wing relative to the Death Star

The acceleration of the X-Wing relative to the Death Star is **2.758 m/s^2** in the positive y direction and **8.147 m/s^2** in the negative z direction. (see MATLAB code above)

d. Comment on the validity of the two-body problem for this situation after the Death Star arrives.

Since the percentage difference between the acceleration between Alderaan and the X-wing is so small when you consider the N-body problem versus the 2-body problem, I'd say that the 2-body problem is valid in this case. The acceleration of the X-wing relative to the Death star is also extremely small. The The two-body problem simplifies the problem and doesn't take as much time.



PART II

Your X-Wing fighter was initially in a circular polar orbit around Alderaan with a radius of 6,400km. Alderaan has a mass of $5x10^{24}$ kg and a planetary radius of 6000km. The X-Wing and Death Star are still in the Z-Y plane as shown in the figure below.

a. Find the initial circular orbit speed, specific mechanical energy and specific angular momentum of the X-Wing around Alderaan.

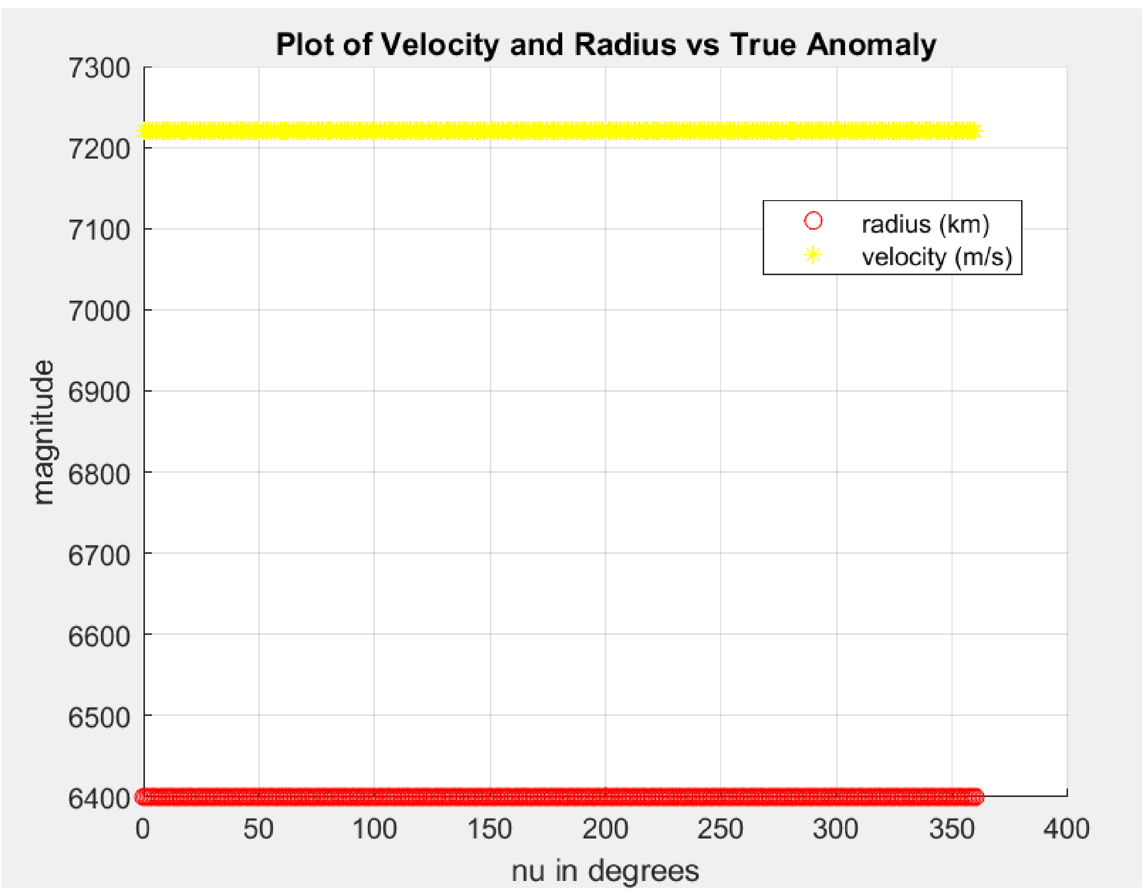
The circular orbit speed is **$7.2208x10^3$ m/s**, the specific mechanical energy is **$-2.6070x10^7$ m^2/s^2** and the specific angular momentum is **$4.6213x10^{10}$ s^{-1}**.

See MATLAB code below:

```
%planetary radius of Alderaan
rp = 6000 * 1000; %m
%radius of x-wing's circular orbit around Alderaan
rx = 6400 * 1000; %m
```

```
%mass of alderaan
ma = 5*10^24; %kg
%mass of x-wing
mx = 10000; %kg
%GM for alderaan
mu_a = G*ma;
%velocity
v_cir = sqrt(mu_a/rx) %m/s
%specific mechanical energy
epsilon = - mu_a/(2*rx)
%flight path angle
phi = 0; %for a circle, deg
%angular momentum
h_ax = rx * v_cir * cosd(phi);
```

b. Using MATLAB, Excel, or Python plot the radius of the orbit as a function of true anomaly. Using the energy equation, plot velocity as a function of true anomaly.
    i. Note: you will use the same methods to plot orbits throughout the project, so make sure the orbital and planet parameters are input as variables.



As you can see, since the orbit is circular, neither radius nor velocity change as a result of a change in true anomaly.

PART III

The Death Star fires its primary weapon and destroys Alderaan, your skills as an X-wing pilot are unmatched and you are able to avoid all the debris from the resulting explosion. The Death Star now becomes the primary body and your X-Wing has a flight path angle ϕ = 10.893° with respect to its new orbit around the Death Star. Assume the debris from Alderaan is gone instantly, but still in the area preventing you from jumping to hyperspace. The Death Star captures you with their tractor beam, the effect of which is a large increase in the Death Stars gravitational parameter, μ_{DS} = 16,685.2 km^3/s^2. The initial orbit conditions are as shown in the figure below.

For this new orbit around the Death Star:

c. Find the eccentricity, velocity (at the initial point shown in the figure), specific mechanical energy, and specific angular momentum.

I assumed that the initial velocity of the X-Wing is the same as it was when in its polar orbit around Alderaan, designated in the MATLAB code as v_cir: **7.2208x10^3 m/s.** Using that and the new gravitational parameter along with the flight path angle to work with, I found that the eccentricity was **38.0713 m,** the specific mechanical energy was **2.4759x10^7 m^2/s^2**, and the specific angular momentum was **4.6213x10^{10} s^{-1}.** See MATLAB code below:

```
%% Project II part 3

mu_ds = 16685.2*10^9; %new gravitational parameter of death star
%radius between death star and x-wing in meters
r_dsx = norm(r31);
%new flight path angle
phi = 10.893; %deg
%angular momentum between death star and x-wing, assuming the velocity is
%the same as before
h_dsx = r_dsx*v_cir*cosd(phi);
%specific mechanical energy between death star and x-wing
epsilon_dsx = v_cir^2/2 - mu_ds/r_dsx
%semi-major axis
a_dsx = - mu_ds / (2*epsilon_dsx)
%semi-lattice rectum
P = h_dsx^2 / mu_ds
%eccentricity e
e  = sqrt (1 - P/a_dsx)
```

d. Find the periapsis and apoapsis radii.

Since the orbit has a positive specific mechanical energy and negative value for semi-major axis, it must be a hyperbolic orbit. Therefore, there is no apoapsis radius. The periapsis radius can be described by a, the semi-major axis, which has an absolute value of **3.3695x10^5 m**. See MATLAB code below:

```
%periapsis radius for hyperbolic orbit
rp = abs(a_dsx)
```

e. Find the periapsis and apoapsis velocities.

Periapsis velocity is **2.6781×10^5 m/s**. There is no apoapsis because the orbit is hyperbolic. However, because it is hyperbolic, we know that as r approaches infinity, velocity will approach $[V_{bo}^2 - 2mu/r_{bo}]^{1/2}$, which is **$7.03669 \times 10^3$ m/s**.

```
%velocity at periapsis m/s
vp = h_dsx/rp
```

```
%velocity as r approaches infinity m/s
v_inf = sqrt(v_cir^2 - (2*mu_ds)/r_dsx)
```

f. Find the escape velocity at periapsis and apoapsis

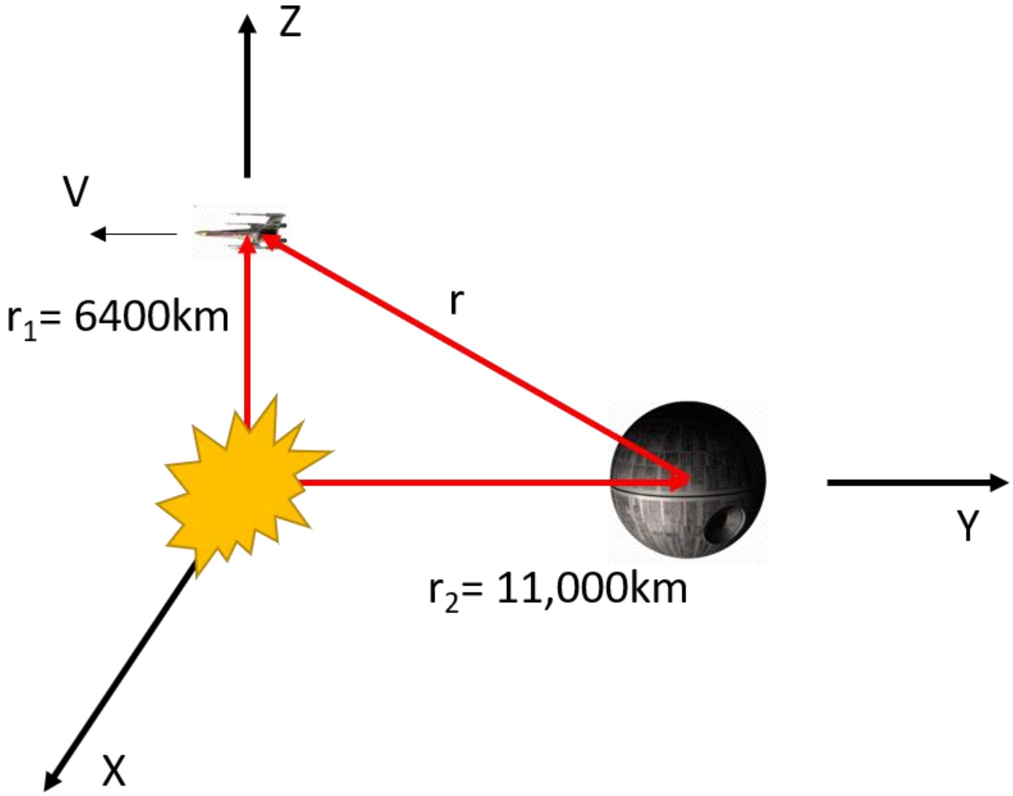
Escape velocity at periapsis is **9.9517×10^3 m/s**. There is no apoapsis because the orbit is hyperbolic.

```
%escape velocity at periapsis m/s
V_esc_p = sqrt(2*mu_ds/rp)
```

g. To try to break free of the tractor beam you increase your throttle to 100% which provides a ΔV = 3 km/s. What is the resulting hyperbolic excess speed with respect to the Death Star, assume you are still at the initial orbit position.

The X-wing is already flying fast enough to surpass escape speed, which is **1.6193×10^3 m/s.** A deltaV is not necessary, and the resulting hyperbolic excess speed is **7.03669×10^3 m/s**, as shown in part e.

```
%escape velocity at current radius m/s
V_esc = sqrt(2*mu_ds/r_dsx)
```



After escaping the Death Star you perform a hyperspace jump and end up in a circular orbit at 400 km in altitude around Tatooine (which remarkably is the same size and weight as Mars). You decide to maneuver into a circular orbit of 100 km.

a. Calculate the deltaV required for this orbital maneuver assuming you use the most efficient maneuver with respect to deltaV

The total deltaV for this orbital maneuver is **0.1415 m/s**. See MATLAB code below.

```
%% Project II part 4

%mass of tatooine
mt = 0.64169*10^24; %kg
mu_t = 0.042828*10^9; %m^3/s^2
r_t = 3389.5*1000; %m volumetric mean radius of Mars, which is very similar to
Tatooine
%radius of orbit
r_p = 400*1000 + r_t; %altitude + radius of tatooine
v_cirt = sqrt(mu_t/r_p) %velocity of transfer orbit @ periapsis
r_a = 100*1000 + r_t; %desired altitude
eps_t = -mu_t / (r_a + r_p); %mechanical energy of transfer orbit
v_p = sqrt(2*(mu_t/r_p + eps_t));
deltav1 = v_cirt - v_p;
v_cirt2 = sqrt(mu_t/r_a);
```

```
v_a = sqrt(2*(mu_t/r_a + eps_t));
deltav2 = v_a - v_cirt2;
%Delta V for Hohmann Transfer
deltav_tot = deltav1 + deltav2; %m/s
```

a. Describe the magnitude and direction (with respect to the spacecraft) of each burn

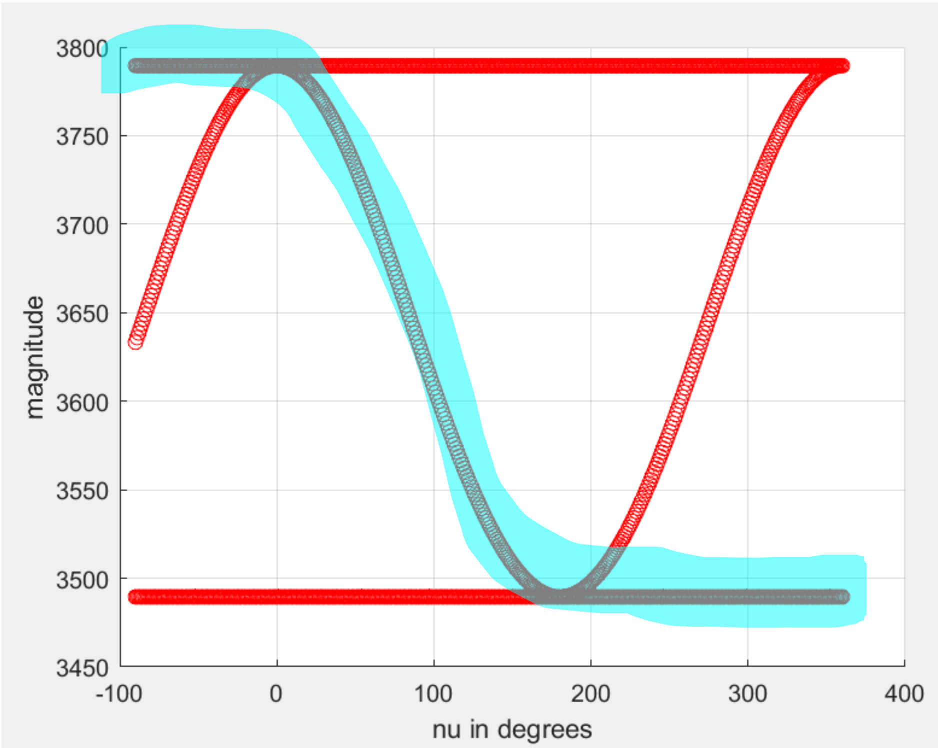
Both deltaV's are positive in magnitude. The first deltaV will be in the opposite direction of motion in order to slow down the circular orbit to reach the transfer orbit. The second deltaV burn will be in the direction of motion in order to re-circularize the orbit.

b. How long do you spend in the transfer orbit?

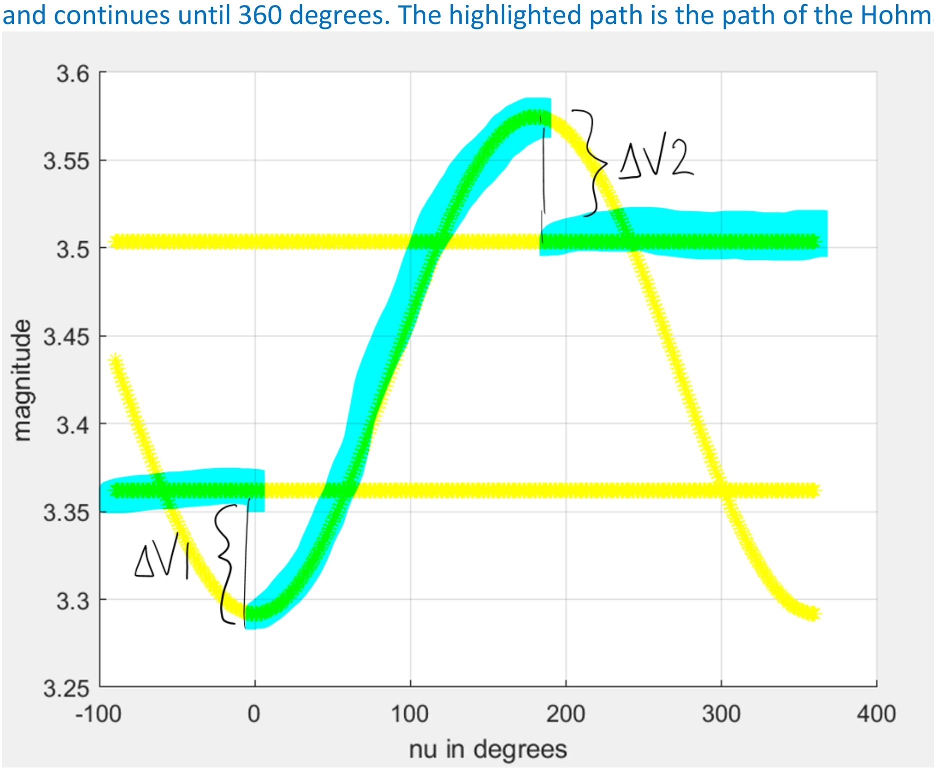
3.331×10^{6} seconds, which is about **38 and a half days.**

```
%semi-major axis of transfer orbit
a_t = (r_p + r_a)/2
%eccentricity of transfer orbit
e_t = r_a/a_t - 1
%time of flight
TOF = pi*sqrt(a_t^3 / mu_t); %sec
```

c. Using the tool you generated earlier plot the radius of the initial circular orbit, transfer orbit, and final circular orbit as a function of true anomaly. Using the energy equation, plot velocity as a function of true anomaly.
    a. Note: you will have to select the true anomaly values to plot for the transfer orbit; it may not be 0-2π.



The above graph shows the radius of orbit with an arbitrary True Anomaly that starts from -90 degrees and continues until 360 degrees. The highlighted path is the path of the Hohmann transfer orbit.



The above shows the velocity of the X-Wing in orbit around Tatooine vs. an arbitrary True Anomaly that starts from -90 degrees and goes to 360 degrees. The highlighted path is the change velocity with respect to the True Anomaly (assuming the vector at which the True Anomaly is 0 is the radius of periapsis of the transfer orbit pointing outward from its focus point. The jumps in the path are due to the first and second deltaV's.

MATLAB code below:

```
nu = [-90:1:360]; %degrees
a = [r_p a_t r_a];
e = [0 e_t 0];
epsilon = [-mu_t/(2*r_p) -mu_t/(2*a_t) -mu_t/(2*r_a)];

hold on
grid on

for k = 1:3

    r = (a(k)*(1 - e(k)^2))./(1 + e(k).*cosd(nu));
    v = sqrt(2 *(mu_t./r + epsilon(k)));
 %   plot(nu,r./1000,'or')
    plot(nu,v,'*y')

end
xlabel('nu in degrees')
ylabel('magnitude')
```